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Higher Dimensional Cosmological Model of the Universe with Decaying Λ Cosmology with Varying *G*

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Abstract In this paper we present higher dimensional cosmological model of the universe with the decaying vacuum energy density in the realm of model with a time varying gravitational constant. We have shown that our model admits the usual higher dimensional de Sitter solution and the other solutions characterized by the constant ratio between matter density and the total energy density. Our work is the generalization of the work obtained earlier by Carneiro (Proceedings of the MG10 Meeting held at Brazilian Center for Research in Physics (CBPF), Rio de Janeiro, Brazil, 20–26 July, 2003) in four dimensional space-time.

Keywords Higher dimensional theory of gravitation · Variable Λ · Gravitational constant *G*

1 Introduction

A revolutionary development seems to have taken place in cosmology during the last few years. The latest development of super-string theory and super-gravitational theory have created interest among scientists to consider higher dimensional space-time, for study of the early universe. A number of authors (Sahadev [1], Emelyanov et al. [2], Chatterjee et al. [3] and [4]) have studied the physics of universe in higher-dimensional space-time. Overdium and Wesson [5] have presented an excellent review of higher-dimensional unified theories, in which the cosmological and astrophysical implifications of extra-dimension have been discussed.

In the Einstein's field equations there are two parameters, the cosmological constant Λ and gravitational constant G. The Newtonian constant of gravitation, G, plays the role of a coupling constant between the geometry and the matter of the Einstein field equations.

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Ozar and Taha ([6] and [7]) have proposed a model in which the cosmological constant Λ is time dependent and the cosmic density ρ equals the Einstein-de Sitter critical density ρ_c. In a separate development, independent of critical density ρ_c assumption, Chen and Wu [8] suggested that Λ is proportional to R^{-2}. They have shown that such a behaviour is deducible from the simple general principle in the line with quantum gravity. Generally a variable cosmological constant implies creation of radiation and matter and non-conservation of entropy [9, 10]. The condition $\rho = \rho_c$, and the requirement of the increasing entropy completely determines Λ in terms of Robertson-Walker scalar factor 'R'. In this paper we consider the two fold energy content, formed by dust matter and energy density ρ_m and by vacuum term with equation of state $p = -\rho_\Lambda$. Then the total energy and pressure are given by $\rho = \rho_m + \rho_\Lambda$ and $p = -\rho_\Lambda$.

One of the most important and outstanding problem in the cosmology is the cosmological constant problem. The recent observations indicate that $\Lambda \sim 10^{-55}$ cm^{-2} while particle physics prediction for Λ is greater than this value by factor of order 10^{120}. This discrepancy is known as cosmological constant problem. Some of the recent discussions on the cosmological constant 'problem' and the consequences on cosmology with matter varying cosmological constant Λ are investigated by Ratra and Peebles [11], Dolgov and co-authors ([12–14]).

There are many cosmological solutions dealing with higher-dimensional models containing a variety of matter fields. However, few work in literature is available where variable G and Λ have been considered in higher dimensions.

In the present paper we considered higher dimensional, R-W model, specially flat, decaying Λ cosmology, in the realm of the model where the gravitational constant G varies with cosmological scale. We show that for late times, such a cosmology is in accordance with the observed value of the cosmological parameter. We also obtained the ratio of the matter density and total energy density.

2 Decaying Λ Solutions with Constant *G*

Let us consider $(n+2)$-dimensional homogeneous, isotropic and flat cosmological model of universe, represented by the space-time of the form

$$ds^2 = dt^2 - R^2(t)\left[dr^2 + r^2(dx_n^2)\right], \tag{1}$$

where $R(t)$ is the scale factor and

$$dx_n^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \cdots + \sin^2\theta_1 \sin^2\theta_2 \cdots \sin^2\theta_{n-1} d\theta_n^2.$$

The energy momentum tensor is given by

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - p g_{\mu\nu},$$

where p is the fluid pressure, and ρ the energy density of the fluid, u_μ is the $(n+2)$-velocity vector such that $u_\mu u^\mu = 1$.

The Einstein field equations in the flat case are given by

$$\frac{n(n+1)}{2}\frac{\dot{R}^2}{R^2} = \rho, \tag{2}$$

$$\frac{n\ddot{R}}{R} + \frac{n(n-1)}{2}\frac{\dot{R}^2}{R^2} = -p. \tag{3}$$

The energy conservation equation in $(n+2)$-dimension is given by

$$\dot{\rho} + (n+1)H(\rho + p) = 0, \tag{4}$$

where n is dimension, $H = \frac{\dot{R}}{R}$ is the Hubble parameter and $8\pi G = 1$.

We consider total energy and pressure of the form

$$\rho = \rho_m + \rho_\Lambda, \tag{5}$$

$$p = -\rho_\Lambda. \tag{6}$$

Now by substitution of (3), (5) and (6) into (4), we get

$$n\dot{H} + \rho_m = 0. \tag{7}$$

Here we have assumed that, for late times, H and ρ_m falls monotonically with the scale factor R. We have expanded them in negative power series of R and by taking leading terms we then have

$$H = \frac{\beta}{R^k}, \tag{8}$$

$$\rho_m = \frac{\gamma}{R^N}, \tag{9}$$

where N and k are positive constants. Substituting in (7) leads to

$$\gamma R^{-N} - n\beta^2 k R^{-2k} = 0. \tag{10}$$

The above equation will be valid for large values of R only when

$$N = 2k, \tag{11}$$

$$\gamma = n\beta^2 k. \tag{12}$$

For the above values, Ω_m becomes

$$\Omega_m = \frac{\rho_m}{\rho} = \frac{2k}{(n+1)}. \tag{13}$$

As $\Omega_m \leq 1$, it follows that $0 \leq k \leq \frac{(n+1)}{2}$.

If $k = 0$, then

$$H = \beta, \tag{14}$$

$$\rho_m = 0. \tag{15}$$

The above solution resembles to Higher dimensional de Sitter Universe, with a constant vacuum energy density.

In case $k = 1$, we get

$$R = \beta t, \tag{16}$$

$$Ht = 1, \tag{17}$$

$$\Omega_m = \frac{\rho_m}{\rho} = \frac{2}{(n+1)}. \tag{18}$$

Jain et al. [15] and Dev et al. [16] have discussed the solutions of the form $R \propto t^N$ in the four dimensional space-time. In our case we get $R \propto t$ and $t = \frac{1}{H}$ which gives the best estimation for age of the universe. The relative matter density is equal to $\frac{2}{(n+1)}$, is not consistent with the observed amount of visible and dark matter.

3 Decaying Λ Solutions with Varying *G*

We consider G as varying gravitational 'constant', then the field equations (2) and (3) can be written as

$$\rho = \frac{n(n+1)H^2}{16\pi G}, \tag{19}$$

$$\frac{n\ddot{R}}{R} + \frac{n(n-1)}{2}\frac{\dot{R}^2}{R^2} = -8\pi G, \tag{20}$$

where $8\pi G$ has been reintroduced.

The continuity (4) does not depend on varying or constant character of G, being just an expression of energy conservation. To solve (4) we consider two different forms of G, discussed in case I and case II.

3.1 Case I

As to the variation law for G, the Eddington-Weinberg empirical Relation Mena Marugan and Carneiro [17]

$$G \approx \frac{H}{m^3} = \frac{H}{8\pi\lambda}, \tag{21}$$

where m has the order of the pion mass, and the constant λ was introduced for convenience. This relation is once again assumed to be valid today or for any late time. Using (21) and (19), we get

$$\rho = \frac{n(n+1)}{2}H\lambda. \tag{22}$$

The total energy and pressure is again assumed in the form of (5) and (6). Hence, the continuity equation (4) can be written as

$$\frac{n\lambda\dot{H}}{2} + H\rho_m = 0. \tag{23}$$

Expanding again H and ρ_m in the negative powers of R, and taking only leading terms, thus by using (8) and (9) we get

$$\gamma R^{-N} - \frac{n\lambda\beta k R^{-k}}{2} = 0. \tag{24}$$

The possible solutions are given by

$$N = k, \tag{25}$$

$$\gamma = \frac{n\lambda\beta N}{2}. \tag{26}$$

For $N=0$, from (8), (9) and (21) we get

$$H=\beta, \tag{27}$$

$$\rho_m=0, \tag{28}$$

$$G=\frac{\beta}{8\pi\lambda}. \tag{29}$$

This solution is usual higher dimensional de Sitter solution, with Λ and G constant.

For the other values of N, from (8), (9) and (19) we obtain

$$R=(N\beta t)^{\frac{1}{N}}, \tag{30}$$

$$Ht=\frac{1}{N}, \tag{31}$$

$$\frac{\rho_m}{\rho}=\frac{N}{(n+1)}. \tag{32}$$

which means that, $N\le(n+1)$.

For $N=1$, we get

$$R=\beta t, \tag{33}$$

$$Ht=1, \tag{34}$$

$$\frac{\rho_m}{\rho}=\frac{1}{(n+1)}. \tag{35}$$

3.2 Case II

Now G is taken as Singh [19]

$$G=\frac{1}{H}. \tag{36}$$

Using (36) and (19), we get

$$\rho=\frac{n(n+1)}{16\pi}H^3. \tag{37}$$

Assuming the total energy and pressure again given by (5) and (6), and using (37), the continuity equation (4) becomes

$$\frac{3nH\dot{H}}{16\pi}+\rho_m=0. \tag{38}$$

Now by using (8) and (9) for H and ρ_m, the above equation can be written as

$$\gamma R^{-N}-\frac{3n\beta^3kR^{-3k}}{16\pi}=0. \tag{39}$$

The possible solutions are given by

$$N=3k, \tag{40}$$

$$\gamma=\frac{n\beta^3N}{16\pi}. \tag{41}$$

For $N=0$ we get similar values for H and ρ_m as discussed in (27) and (28) except G, which is given by

$$G=\frac{1}{\beta}. \tag{42}$$

For the other values of N, from (8), (9) and (37) we obtain

$$R=\left(\frac{N\beta t}{3}\right)^{\frac{3}{N}}, \tag{43}$$

$$Ht=\frac{3}{N}, \tag{44}$$

$$\frac{\rho_m}{\rho}=\frac{N}{(n+1)}. \tag{45}$$

For $N=1$, we get

$$R=\left[\frac{\beta t}{3}\right]^3, \tag{46}$$

$$Ht=3, \tag{47}$$

$$\frac{\rho_m}{\rho}=\frac{1}{(n+1)}. \tag{48}$$

4 Conclusion

This work has thus generalized to higher dimension the well known results in four dimension. It is found that there may be significant differences in the principle at least, from the analogous situation in four dimensional space-time. In this work we have analysed higher dimensional Robertson-Walker cosmological model with decaying vacuum energy density in the presence of varying gravitational constant 'G' of the form $G\propto H$ and $G\propto\frac{1}{H}$. We have obtained the variation rate of G and the rate of matter production.

For case I, we used the evolution law

$$G=\frac{H}{8\pi\lambda}, \tag{49}$$

which leads to the relative variation rate

$$\frac{\dot{G}}{G}=-(1+q)H, \tag{50}$$

where $q=-\frac{R\ddot{R}}{\dot{R}^2}$, is the deceleration parameter.

For $N=1$, we have $q=0$, and therefore

$$\frac{\dot{G}}{G}=-H. \tag{51}$$

For the matter production (coming from the decaying vacuum energy), we have for ($N=1$)

$$\frac{1}{R^{n+1}}\frac{d}{dt}(\rho_m R^{n+1})=n\rho_m H. \tag{52}$$

Now, the vacuum energy density and the corresponding cosmological 'constant' are given by

$$\rho_\Lambda = \left[\frac{n[(n+1)-N]}{16\pi}\right] m^3 H, \tag{53}$$

$$\Lambda = \left[\frac{n[(n+1)-N]}{2}\right] H^2. \tag{54}$$

For case II, we used (36) which leads to the relative variation rate

$$\frac{\dot{G}}{G} = (1+q)H. \tag{55}$$

For $N = 1$, we have $q = -2/3$, and therefore

$$\frac{\dot{G}}{G} = \frac{H}{3}. \tag{56}$$

For the matter production (coming from the decaying vacuum energy) for $N = 1$, we get equation similar to (52). Now, the vacuum energy density is given by

$$\rho_\Lambda = \left[\frac{n[(n+1)-N]}{16\pi}\right] H^3, \tag{57}$$

where as the corresponding cosmological 'constant' coincides with the one obtained in case I. Similarly, it is observed that the deceleration parameter $q = 0$, but in case II, q is negative i.e. the expansion of the universe is possible. It is also observed that in both the cases, for $N = 0$, the Hubble parameter and the matter density are equal and G is in terms of β. Similarly the cosmological constant Λ comes out to be same in both the cases for $N = 1$. For $n = 2$ our results are similar to the results obtained earlier by Carneiro [18] for the case $G \sim \frac{H}{m^3}$.

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